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SIGMA-ZED: A COMPUTATIONALLY EFFICIENT APPROACH TO REDUCE THE HORIZONTAL GRADIENT ERROR IN THE EFDC'S VERTICAL SIGMA GRID

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The Environmental Fluid Dynamics Code (EFDC), originally developed at Virginia Institute of Marine Science, is a well-known and widely applied three dimensional hydrodynamic model used for many environmental applications (Hamrick, 1996). In the horizontal, the EFDC numerical scheme uses an orthogonal curvilinear grid. In the vertical, EFDC uses a sigma coordinate transformation that uses the same number of layers for all cells in the domain. To accommodate the varying depths over the model domain, the thickness of the layers vary from cell to cell but the number of layers and fraction of depth for each layer are constant. This approach introduces a well-known error in the density gradient terms, otherwise known as the pressure gradient error (Mellor, et.al. 1994). These errors are most pronounced in regions with steeply varying bathymetry. A new vertical layering approach that is computationally efficient has been developed and applied to the EFDC model, thereby reducing these pressure gradient errors. The vertical layering scheme has been modified to allow for the number of layers to vary over the model domain. Each cell can use a different number of layers, though the number of layers for each cell is constant in time. As with the Sigma stretch approach, the thickness of each layer varies in time to accommodate the time varying depths. The z coordinate system varies for each cell face, matching the number of active layers to the adjacent cells. The new version is computationally more efficient than a similarly configured sigma stretch grid, thus making models with 20 to 50 layers or more practical for typical projects. This new approach has been tested with several hypothetical test cases. The model has been applied to Lake Washington (Seattle, WA, USA), which has steep bottom gradients and sharp thermoclines. The results indicate that the vertical variation of temperature and the thermal stratification are more accurately reproduced and provide a significant improvement compared to the earlier sigma coordinate transformation method.

1. Introduction

The Environmental Fluid Dynamics Code (EFDC) model was originally developed at the Virginia Institute of Marine Science for simulation on three dimensional flow, transport, and biogeochemical processes in surface water systems including rivers, lakes, estuaries, reservoirs, wetlands, and near-shore to shelf-scale coastal regions (Hamrick, 1992a, 1992b and 1996). In addition to hydrodynamic and salinity and temperature transport simulation capabilities, EFDC is capable of simulating cohesive and non-cohesive sediment transport, near-field and far-field discharge dilution from multiple sources, the transport and fate of toxic contaminants in the water and sediment phases, and the dissolved oxygen/nutrient process (i.e. eutrophication). Special enhancements to the hydrodynamics of the code, including vegetation resistance, drying and wetting, hydraulic structure representation, Lagrangian particle tracking (Chung and Craig, 2012), wave current boundary layer interaction, and wave-induced currents, allow refined modeling of wetland and marsh systems, controlled-flow systems, and near-shore wave-induced currents and sediment transport. Parallelization of the EFDC code for open multi-processing has also been performed (Craig and Chung, 2012).

The orthogonal curvilinear grid is used for the horizontal direction and a transformation of sigma coordinate is used for the vertical direction, in which the number of layers is constant for the whole computational domain. This approach introduces a well-known error in the horizontal gradient terms including the concentration, velocity and the pressure (Mellor, et.al. 1994). In general this error only occurs in regions with steeply varying bathymetry. In order to overcome this weakness, a new vertical layering approach that is computationally efficient has been developed and applied to the EFDC model. The vertical layering scheme has been modified to allow the number of layers to vary over the model domain based on the water depth. Consequently, each cell can use a different number of layers. The z coordinate system varies for each cell face, matching the number of active layers to the adjacent cells. Such a transformation is called as sigma-zed (SGZ) coordinate. In addition, in case of significant bed level change due to erosion/deposition, the number of layers at each cell is re-determined for each time step. It should be noted that in SGZ coordinate the number of vertical layers can be very large, but the computational time is shorter in comparison with a similarly configured sigma (SIG) coordinate model.

In this paper the new algorithm on vertical layering is developed to improve EFDC model and applied to Lake Washington (Seattle, WA, USA). Lake Washington is characterized by steep bottom gradients and sharp thermoclines, features well suited to demonstrating the advantages of the SGZ coordinate system.

2. Algorithm modification in EFDC with SGZ

The governing equations of EFDC include Navier-Stokes for fluid flow, the advection-diffusion equations for salinity, temperature, dye, toxicants, constituents in water quality and suspended sediment transport (Hamrick, 1992, 1996; Hamrick and Wu, 1997). In the horizontal direction, the equations are presented in the curvi-linear coordinate system and Sigma transformation ($z = 0$ at bed and $z = 1$ at water surface) for the vertical direction. They are discretized with the finite difference method based on an explicit scheme. A detailed description of the EFDC algorithm exceeds the scope of this paper. For SGZ transformation the equations are still the same, however, the number of layers at each cell differs based on a factor determined on the basis of the ratio between bed elevation and the minimum elevation. In addition, the thickness of layers at each cell has to satisfy the following expression:

$$\sum_{k=n}^{K} \Delta z_k = 1 \qquad (1)$$

in which K is the maximum number of layers, n the index of bed layer and Δz_k the thickness of layer k.

For the original sigma the index of the bed layer always is equal to 1 while in the SGZ this value can be varied in the range $1 \leq n \leq K$ depending on the number of layers due to the rescaling. This requirement improves the accuracy of the horizontal gradient calculation for the variable $c(L, k)$ of the cell L at layer k:

$$\frac{\partial c(L,k)}{\partial x} = \frac{c(L,k) - c(L-1,k)}{\Delta x(L)} \qquad (2)$$

When sediment transport is simulated and bed morphology is taken into account, the determination of the new indices of bed layers should be implemented at every time step. This is because at this time the ratios between water depths and the maximum are changing due to erosion or deposition in comparison with the initial time. Figure 1 is given as an illustration on the necessity of layering for every time step in case of bed level change due to sedimentation. It is clear that a period of time, such as 10 days for this example, the bed elevations at the cells L-1 and L changed considerably, and at that time the numbers of layers at these cells become unsuitable for the ratio on the initial layers as mentioned above. Therefore, an update of layering for the whole domain is important and necessary for SGZ. However, the re-layering is only an optional approach.

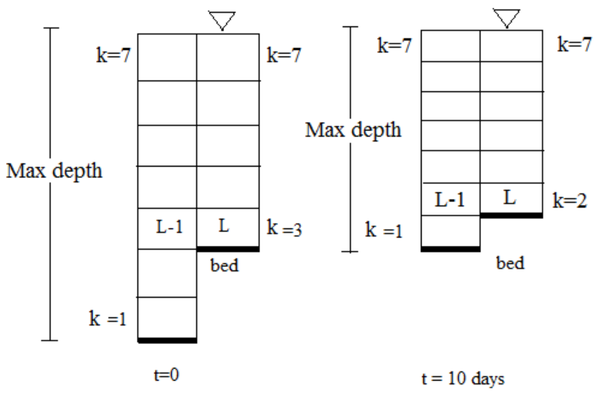

Figure 1. An illustration of SGZ layering before and after a 10 day period with sediment deposition.

The other necessary modification for SGZ coordinate system is the treatment on wet/dry in 3-D calculation of the horizontal gradient when the number of layers at cell L-1 is less than that at cell L. It should be noted that this problem does not appear for SIG coordinates, because the number of layers is the same for every cell. This means that the SIG model requires more calculation time and is therefore one of the weak points of the SIG coordinate system which is overcome with SGZ.

3. Application of EFDC with SGZ to Lake Washington

3.1. *Setting up Model*

Lake Washington is a large freshwater lake adjacent to the city of Seattle and the largest in King County. It is a ribbon lake, 35 km long and surface area of approximately 88 km^2. Mercer Island lies at the southern end of the lake. The main inflows are from the Sammamish River in the north and Cedar River in the south. The outflow is to via a ship canal at Puget Sound in the west with discharge of approximately 7 m^3/s.

The bathymetry of Lake Washington (Figure 2) shows that it is deep, narrow, glacial trough with steeply sloping sides and deepest point approximately 65 m of depth. These morphologic features are highly suitable for application of the SGZ model.

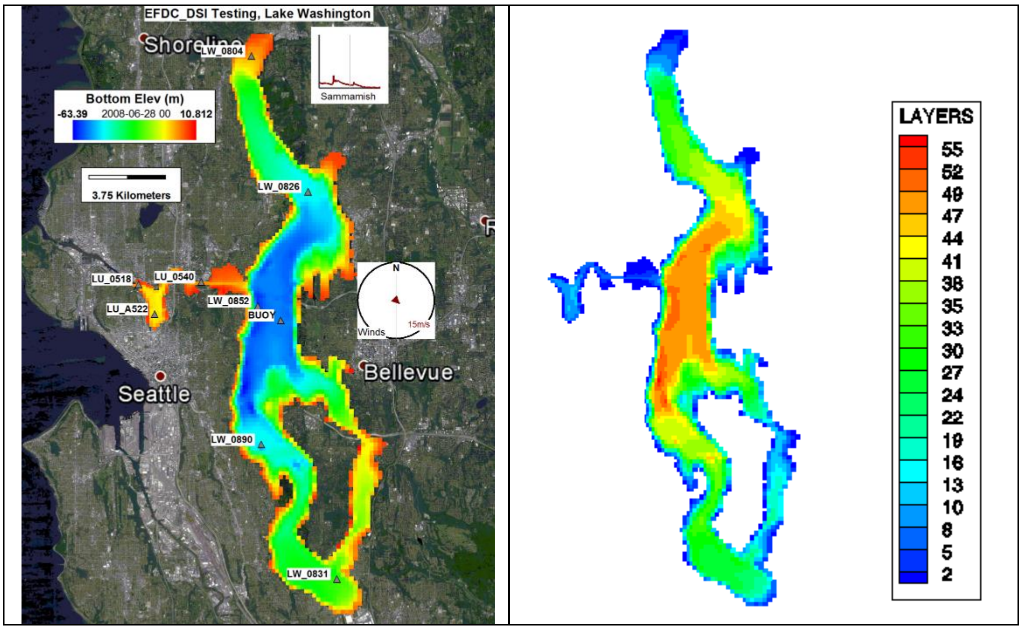

Figure 2. Bathymetry of Lake Washington and number of layers for sigma-zed.

The computational domain comprises a horizontal curvilinear grid of 3,185 cells, in which the range of grid size is from 100 to 200 m. The vertical direction is divided into 55 layers to provide sufficient discretization to investigate the effect of vertical density stratification due to vertical temperature gradients. The SIG layering model includes 175,175 3D computational cells, whereas the SGZ model uses a considerably reduced computational cell count of 83,150. Both models simulated the temperature with surface and bottom heat exchange activated.

The boundary conditions are water discharge and temperatures at six inflow and one outflow boundaries. The two major inflow series located near the north and south regions of Lake Washington are presented in Figure 3 and Figure 4. The meteorological and atmospheric data such as wind speed, wind direction, relative humidity, air temperature and solar radiation were obtained from the King County web site and processed for the EFDC application. The initial temperature structure of Lake Washington was obtained from the Lake Washington buoy data. A water quality monitoring buoy, operated by King County Lake Monitoring Program (https://green.kingcounty.gov/lake-buoy/default.aspx), collects several vertical profiles each day for a range of constituents. The location of the buoy is shown on Figure 2. An example output of one of the vertical profiles for the Lake Washington Buoy is shown in Figure 5. The two EFDC options, original sigma (SIG) and sigma-zed (SGZ), were applied for the same model conditions. Some parameters in the model for the two options are presented in Table 1.

Table 1. Input parameters of two options.

Options	Time (Day)	Time step (Sec)	Layers	Heat
SIG	120	5	55	Yes
SGZ	120	5	2-55	Yes

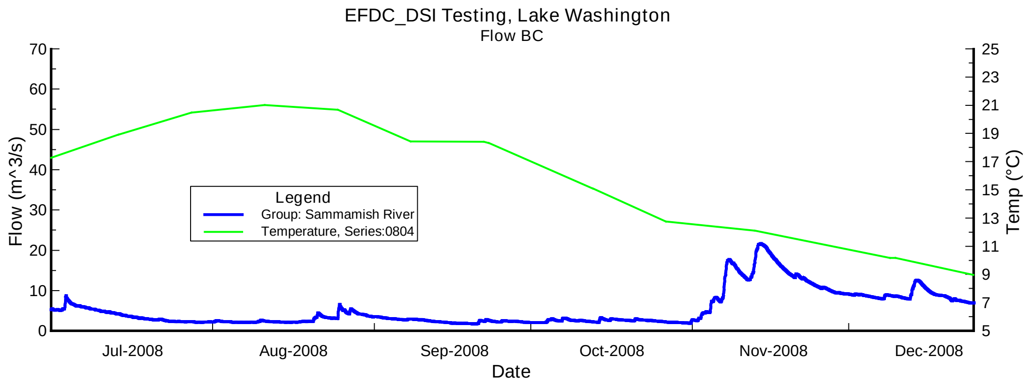

Figure 3. Time series of flow and temperature at Sammamish River.

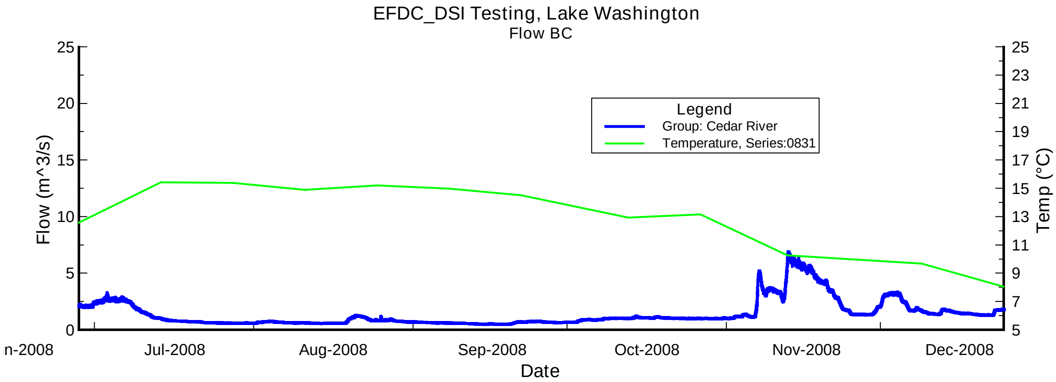

Figure 4. Time series of flow and temperature at Cedar River.

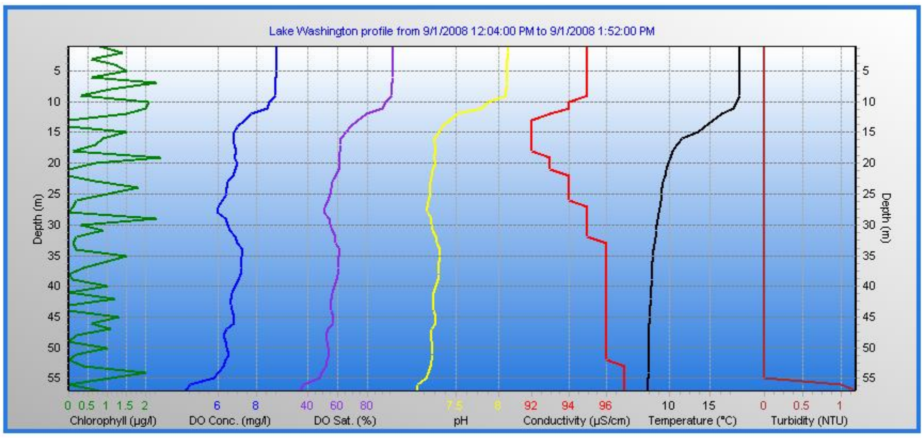

Figure 5. Measurement for Lake Washington profile (from King County Lake Monitoring Program).

3.2. *Simulation Results and Discussion*

The results of simulation over 120 days for both the cases of SIG and SGZ are presented in Figure 6-11. Figure 6 gives a quick visual assessment on the velocity magnitudes at the bottom layer, where it is seen that the velocity field for SGZ is not as strong as that for SIG model. At the same time, the horizontal distributions of velocity magnitude are also quite different between the two cases despite identical initial and boundary conditions. The discrepancy between two results is due to the rescaling the water depth to determine the bottom layer at each cell in the computation domain. In SGZ model the bottom layer is not always on the same layer (k=1) as in SIG, therefore it impacts on the vertical structure of water column and influences the hydrodynamics field on water surface as seen in Figure 7. It also can be inferred that the local adjustment of horizontal gradient terms plays an important role in changing the general behavior of flow (i.e. circulation) between the two models.

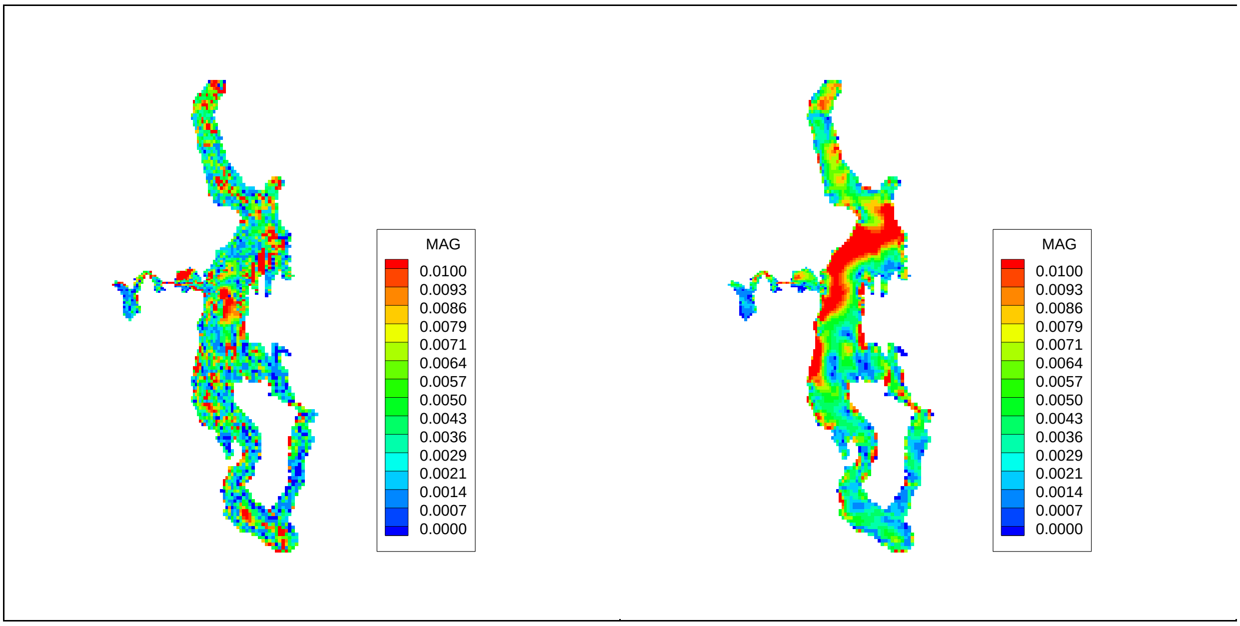

Figure 6. An example time snapshot of the horizontal distributions of velocity magnitude for the bottom layers (Left: SGZ; Right: SIG).

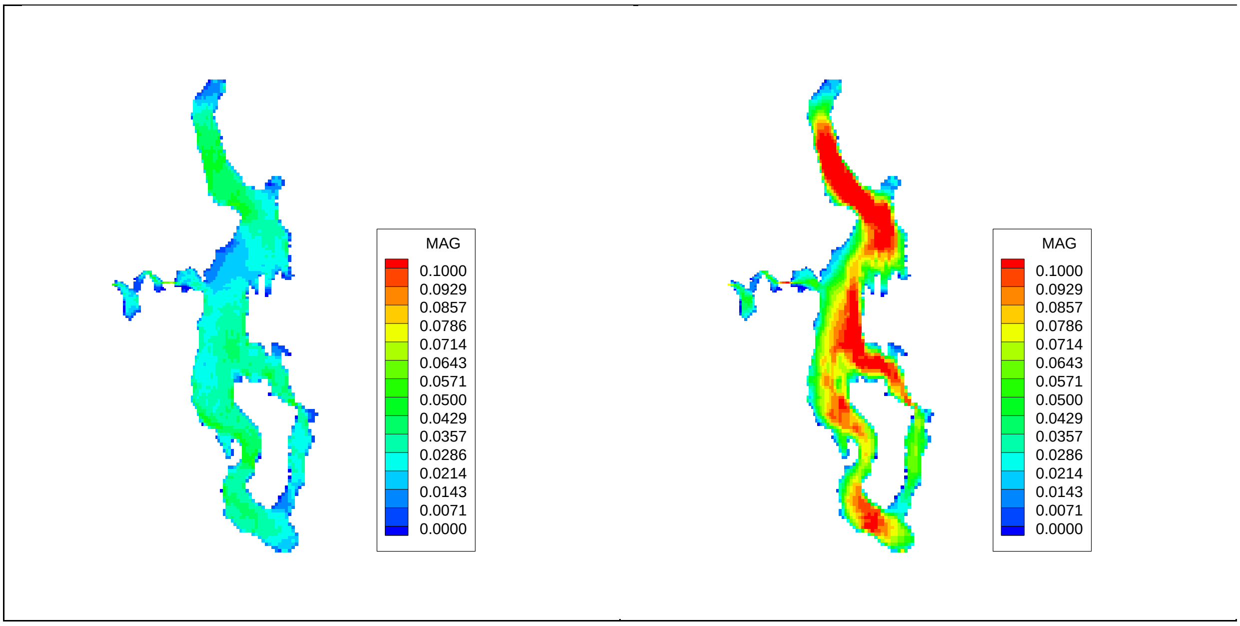

Figure 7. An example time snapshot of the horizontal distributions of velocity magnitude for the top layer (Left: SGZ; Right: SIG).

In order to investigate the influence of horizontal gradient terms on the vertical profiles of temperature as well as the accuracy between SIG and SGZ models, comparisons the models versus the data were computed for every day during the summer 2008 simulation period. Eight representative vertical profiles, starting at the beginning of the simulation and ending on October 15, are presented in Figure 8 for the SIG model and Figure 9 for the SGZ model. By inspection, it is clear that the SGZ model produces a much better representation of the temperature structure of Lake Washington than does the SIG model. While the overall top and bottom temperatures for both models are similar to the data, the SGZ represents the vertical structure much better.

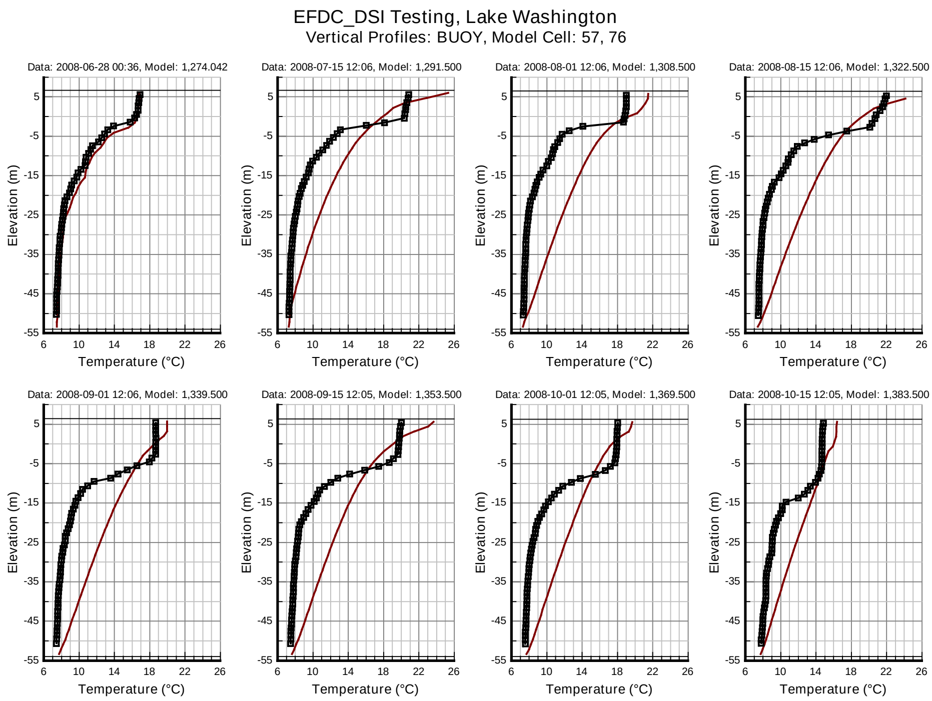

Figure 8. Comparison on vertical profile of temperature between data and computation by SIG model.

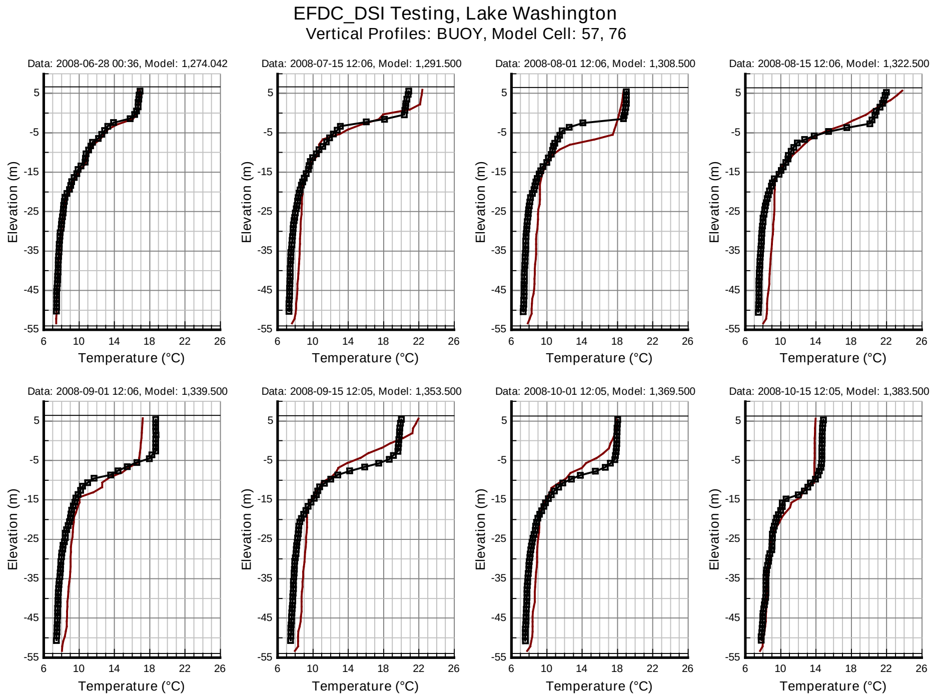

Figure 9. Comparison on vertical profile of temperature between data and computation by SGZ model.

In addition, the time required for running model is one of the strong points of SGZ, especially when the number of layers is large. Some typical runtime statistics for EFDC sub-models are shown in the Table 2. These are the main hydrodynamic module HDMT, the calculation of the 3D velocity field in UVW, the explicit solution, turbulence, heat sub-model and the total elapsed time. It is interesting to see that the time to run EFDC using SIG is 29.1 hours while the time for SGZ is only 25.1, i.e. four hours shorter.

Table 2. Time consumed in hours for both cases.

Options	HDMT	UVW	Explicit	Turbulence	Heat	Elapsed time
SIG	28.2	1.7	3.3	4.4	0.7	29.1
SGZ	24.9	3.4	2.7	3.2	0.5	25.1

4. Conclusions

A modification of the vertical coordinate system algorithm in EFDC with SGZ transformation has been successfully implemented. The new EFDC_SGZ has been tested for several different hydrodynamic regimes. In this paper, the model has been applied to Lake Washington (Seattle, WA, USA), which has steep bottom gradients and sharp thermoclines. A detailed investigation of the velocity and temperature fields has been implemented with the following summary:

- Calculation of the horizontal gradient terms for Navier-Stokes, as well as for advection-diffusion equations in SIG coordinates, are very sensitive to water body bottoms with very steep slopes, therefore SGZ transformation is the recommended choice for such cases.
- Through the comparisons of the computed results of the two models with measured data at the Lake Washington buoy, it can be seen that EFDC with SGZ is more accurate and can better simulate a density stratified regime. While tested with density stratification due to temperature gradients, the SGZ approach should work similarly for stratification do to salinity, such as commonly occurs in estuary salt wedges.
- Computational time for SGZ model is significantly reduced in comparison with SIG model, especially when the number of layers is large. This makes the application of SGZ to real world environmental hydrodynamics practical.
- Finally, the new vertical algorithm using SGZ system in EFDC considerably improved the accuracy of EFDC for representing systems with sharp density gradients.

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